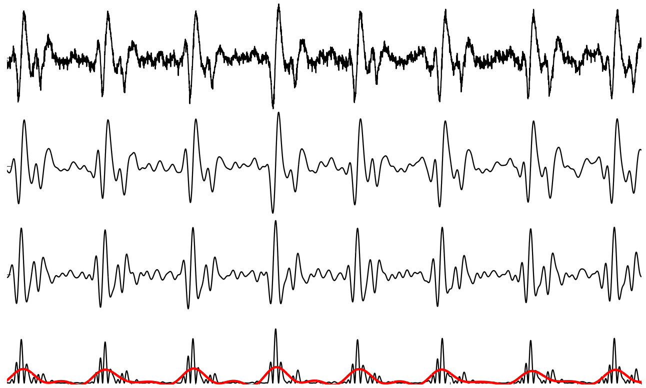
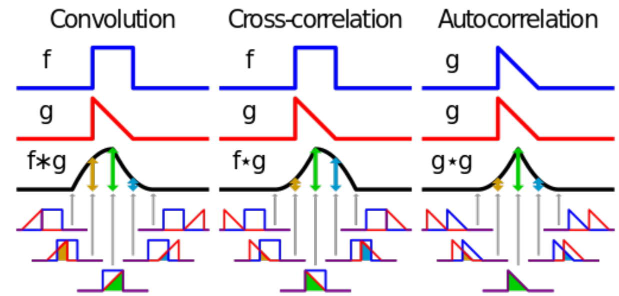
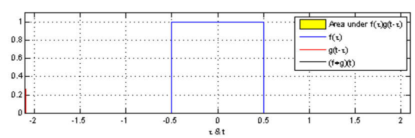
Signal Processing - PRE

Acquisition, Filtering, Sampling & Quantization

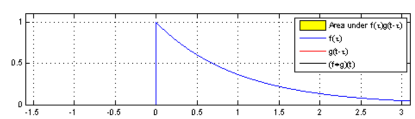


Convolution Revisited





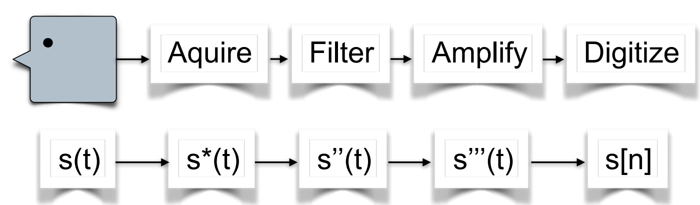
https://commons.wikimedia.org/wiki/File:Convolution_of_box_signal_with_itself2.gif



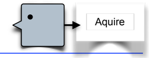
https://commons.wikimedia.org/wiki/File:Convolution_of_spiky_function_with_box2.gif

Steps

- Acquiring The Signal
- Filtering The Signal
- Amplifying a signal
- Sampling an Analog Signal & Quantizing The Signal



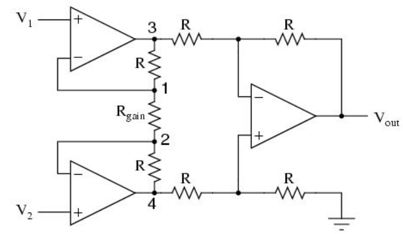
Acquisition



- The goal is to measure the signal - **s(t)** without altering it.
  - Think about measuring voltage and current in a circuit. Where and how are both important.
- Let's use EEG as an example, how can you measure the microvolt differences on the scalp without disturbing system? That is, how is it possible to avoid the having the head serve as current source in the data acquisition?

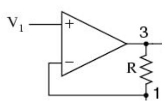
Acquisition

- Recall the Instrumentation Amplifier



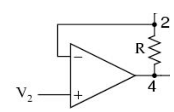
Acquisition

- Which Op-Amp Circuit Does this look like? What does this portion of the circuit do?



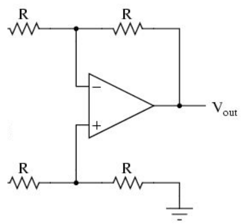
Acquisition

- And This One? What does this portion of the circuit do?



Acquisition

- And this one? What does this portion of the circuit do?



Acquisition

- Beyond just measuring the signal it is also important to do so in a fashion the minimizes the noise. Where possible it is better to avoid or minimize the inclusion noise in the signal
  - Examples of doing this in EEG include:
    - Minimizing contact impedance
    - Minimizing contact movement with respect to the scalp
    - Common mode rejection
    - Driven feedback signal
    - Cable Shielding

Acquisition

$$s(t) = \text{TrueSignal}$$

$$s^*(t) = s(t) + \text{Error}(t) + \varepsilon_N(t) = \text{Measured Signal}$$

$$s^*(t) = s(t) + \varepsilon(t)$$

Filtering

Sampling & Filtering - Spectral Repeats

- Assumption 1: The sampled signal is band limited to some frequency (B)
- Assumption 2: The extent of the band is know

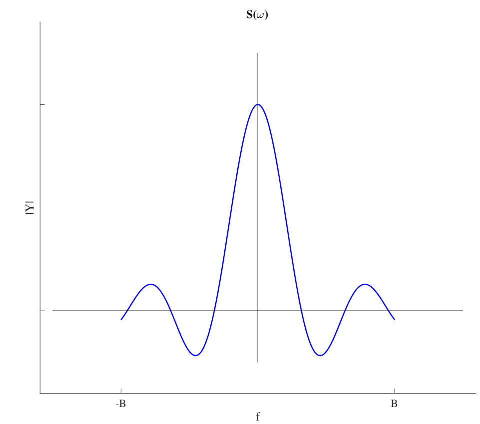
Signal: $s(t)$

Sample Points: $\delta(t - mT_{sample})$, m int, T_{sample} = sample period

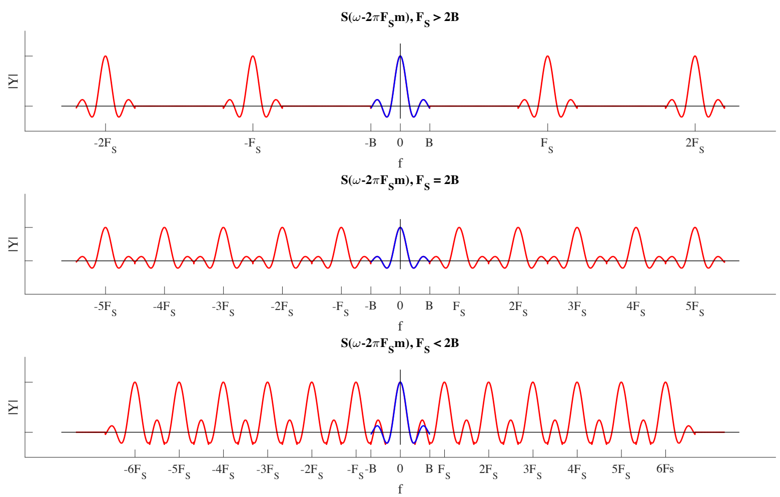
Sampled Signal = $s(t) \times \delta(t - mT_{sample})$

$$s(t)\delta(t - mT_{sample}) \xleftrightarrow{FT} S(\omega) * \delta\left(\omega - \frac{2\pi m}{T_{sample}}\right) = S(\omega - \frac{2\pi m}{T_{sample}})$$

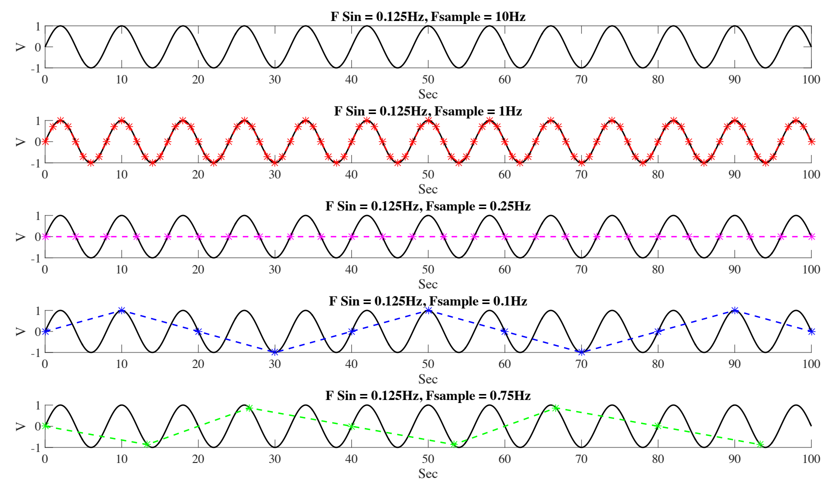
Sampling & Filtering - Nyquist-Shannon



Sampling & Filtering - Nyquist-Shannon



Sampling & Filtering - Nyquist-Shannon



Sampling & Filtering - Spectral Repeats

Signal: $s(t)$

Sample Points: $\delta(t - mT_{sample})$, m int, T_{sample} = sample period

Sampled Signal = $s(t) \times \delta(t - mT_{sample})$

$$F_{sample} = \frac{1}{T_{sample}}$$

$$s(t)\delta(t - mT_{sample}) \xleftrightarrow{FT} S(\omega) * \delta\left(\omega - \tfrac{2\pi m}{T_{sample}}\right) = S(\omega - \tfrac{2\pi m}{T_{sample}}) = S(\omega - 2\pi m F_{sample})$$

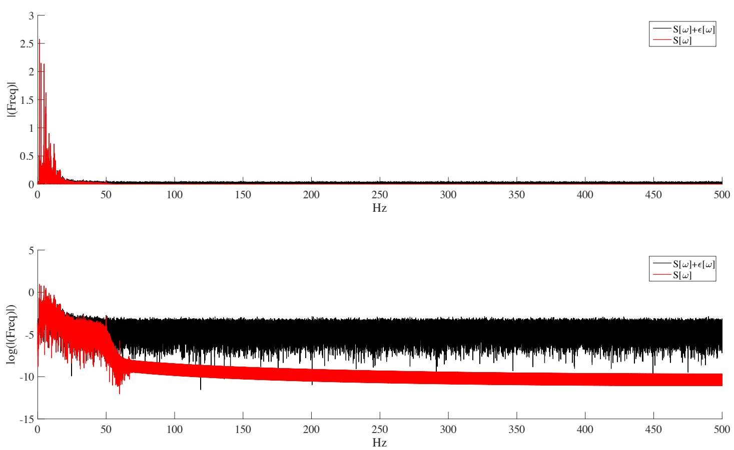
Bandlimit of Signal: B in (Hz)

$$F_{sample} > 2B$$

Sampling & Filtering - Anti-Aliasing Filter

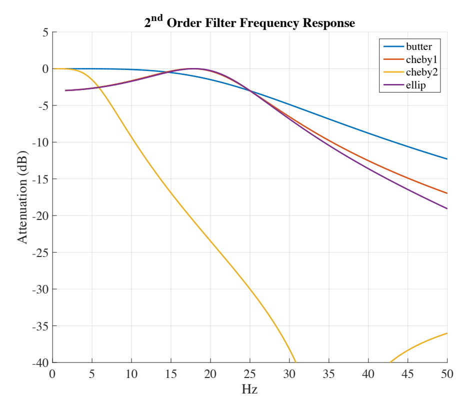
- Yah yah, thats all well and good, but what if my signal is not band limited or you don't know or need the full bandwidth of the signal
- Is noise band limited? Isn't that part of the sampled signal as well?
- How should we handle this?

Sampling & Filtering - Anti-Aliasing Filter



Sampling & Filtering - Anti-Aliasing Filter

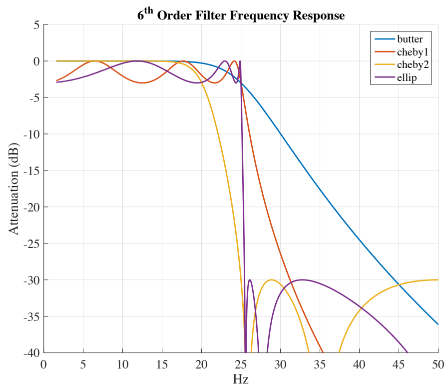
- Filter Type / Shape



$$dB = 10 \times \log_{10}\left(\frac{Power_{in}}{Power_{out}}\right) = 10 \times \log_{10}\left(\frac{Voltage_{in}^{2}}{Voltage_{out}^{2}}\right) = 10 \times \log_{10}\left(\frac{Voltage_{in}}{Voltage_{out}}\right)^{2} = 20 \times \log_{10}\left(\frac{Voltage_{in}}{Voltage_{out}}\right)$$

Sampling & Filtering - Anti-Aliasing Filter

- Filter Type / Shape



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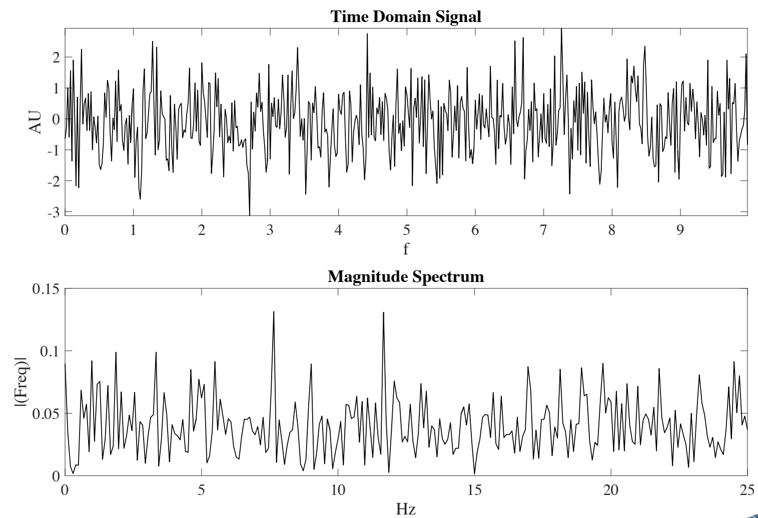
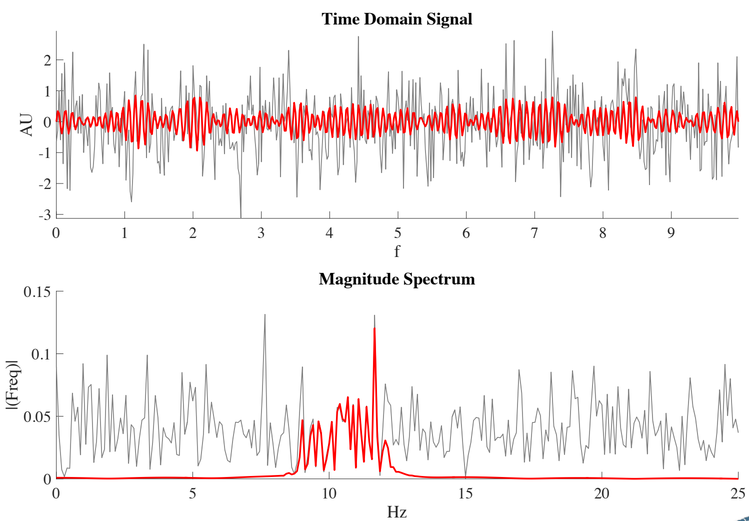


Filters - The Danger of Assumptions



Z-Transform - The Discrete Laplace (sort of)

$$X[z] = Z\{x[n]\} = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

$$z = Ae^{j\phi} = A(\cos\phi + j\sin\phi)$$

Laplace Transform	Z-Transform
$x(t) \leftrightarrow X(s)$	$x[n] \leftrightarrow X[Z]$
$x(t-m)u(t-m) \leftrightarrow e^{-sm}X(s)$	$x[n-m] \leftrightarrow z^{-m}X[Z]$
...	...
https://en.wikipedia.org/wiki/Laplace_transform	https://en.wikipedia.org/wiki/Z-transform

Z-Transform - Causal Filter

$$y[n] = \alpha_0 x[n] + \alpha_1 x[n-1] + \alpha_2 x[n-2] + \ldots$$
$$Y[z] = \alpha_0 X[z] + \alpha_1 z^{-1} X[z] + \alpha_2 z^{-2} X[z] + \ldots$$
$$y[n] = x[n] \otimes h[n] \rightarrow Y[z] = X[x]H[z]$$
$$H[z] = \frac{Y[z]}{X[z]} = \alpha_0 + \alpha_1 z^{-1} + \alpha_2 z^{-2} + \ldots$$
$$z = e^{j\omega}$$
$$H[e^{j\omega}] = \alpha_0 + \alpha_1 \cos(j\omega) - \alpha_1 j \sin(j\omega) + \alpha_2 \cos(j2\omega) - \alpha_2 j \sin(j2\omega) + \ldots$$
$$\left|H[e^{j\omega}]\right| = \sqrt{H[e^{j\omega}]^2}$$
$$\measuredangle H[e^{j\omega}] = \arctan\left(\Im(H[e^{j\omega}]) / \Re(H[e^{j\omega}])\right)$$

Z-Transform - Centered Filter

$$y[n] = \alpha_0 x[n] + \alpha_1 x[n-1] + \alpha_1 x[n+1] + \alpha_2 x[n-2] + \alpha_2 x[n+2] \ldots$$
$$Y[z] = \alpha_0 X[z] + \alpha_1 z^{-1} X[z] + \alpha_1 z^{1} X[z] + \alpha_2 z^{-2} X[z] + \alpha_2 z^{2} X[z] + \ldots$$
$$y[n] = x[n] \otimes h[n] \rightarrow Y[z] = X[x]H[z]$$
$$H[z] = \frac{Y[z]}{X[z]} = \alpha_0 + \alpha_1 z^{-1} + \alpha_2 z^{1} + \alpha_1 z^{-2} + \alpha_2 z^{2} + \ldots$$
$$z = e^{j\omega}$$
$$H[e^{j\omega}] = \alpha_0 + \alpha_1 \cos(j\omega) + \alpha_2 \cos(j2\omega) + \ldots$$
$$\left|H[e^{j\omega}]\right| = \sqrt{H[e^{j\omega}]^2}$$
$$\measuredangle H[e^{j\omega}] = \arctan\left(\Im(H[e^{j\omega}]) / \Re(H[e^{j\omega}])\right) = 0$$

Phase Shift